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# THE CONCEPT OF THE INFINITE AND THE CRISIS IN MODERN PHYSICS

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***Abstract***

The basic thesis is that the problem of infinity underlies the current dilemma in modern theoretical physics. The traditional and set-theoretic conceptions of infinity are considered. It is demonstrated that standard mathematical analysis is dependent on the complete *relativity* of the infinite. In examining the domains of modern physics, infinity is found to lose its entirely relative character and, therefore, to be less amenable to classical analysis. Complementary aspects of microworld infinity are identified and are associated with the equivalent features (inertial and gravitational mass) of Einstein's macroworld theory. The persisting effort to treat essentially non-classical phenomena in classical terms is critically discussed. A new attitude toward the infinite is recommended, one that might lead to establishing a *second* principle of the relativity of the infinite. The prospect for implementing the suggested approach through a "transanalytic" meta-theory of dimensional generation is briefly entertained.

## INTRODUCTION

This essay is predicated on the assumption that the conceptual difficulties besetting modern theoretical physics are serious enough to warrant a thoroughgoing re-examination of the foundations. I intend to show that the idea of infinity is central to these difficulties. My effort is motivated by the conviction that we must clarify the role of the infinite in scientific analysis if we are to come to grips with the challenge that faces us.

## 1. INFINITY: OPEN AND CLOSED

The oldest notion of infinity is the one still held by the conventional wisdom. An infinite number or amount is one that is indefinitely great. Another term is always available beyond the last term counted in such an infinite sequence. This kind of infinity may therefore be called an *open* or *potential* infinity. Almost three hundred years ago, with the advent of the calculus, the method for *closing* or *actualizing* the infinite appeared to have been produced. But in the last century, the development of set theory led to a clarification of the concepts of convergence to a limit and actual infinity.

According to set theory, if we are to understand how infinity can be actualized, we must revise our thinking about groupings of elements. An element is considered a member of a group if it possesses certain characteristics and not others. By shifting our attention to the characteristics themselves and away from the elements characterized, we form an understanding of the group which transcends any particular experiences we might have had with its individual members. As Kramer illustrated, "We can speak of 'Americans' without having a personal acquaintance with each citizen of the United States."(1) This transition from the concrete, open-ended operation of counting elements one by one to conceiving them in the more abstract context of membership in a class or set enables us to appreciate actual infinity. Though we cannot count the members of an infinite set, we can still confirm that the set is infinite by verifying that it possesses a certain characteristic, namely that it can be made equivalent to a proper subset of itself. (This quality is demonstrated if the well known principle of one-to-one correspondence between set and subset is satisfied.)

Then actualization of the infinite clearly entails a *qualitative* transformation. In the words of Georg Cantor (nineteenth century set theorist and innovator of the idea of the transfinite), the goal of the infinite is obtained by "stepping out of the series".(2) Any attempt to build up an infinity from below by mere iteration will be an exercise in futility.

How did Cantor specifically deal with the problem of achieving the infinite? The sequence of the positive real whole numbers: 1, 2, 3, . . ., $v$, . . ., is generated by:

> repeated positing and uniting of basic units … The addition of a unit to an existing, already formed number. I call this … the *first principle of generation.* The number … of the numbers $v$ of class (I) to be formed in this way is infinite, and there is no greatest one among them. As contradictory as it would be, therefore, to speak of a greatest number of class (I), there is, on the other hand, nothing objectionable in conceiving of a *new* number -- we shall call it $\omega$ -- which is intended to be the expression for the fact that the totality (I) as a whole be given in its natural and lawful succession (similar to the way in which $v$ is an expression for the fact that a certain finite number of units is unified into a whole)... if any definite succession of defined whole real numbers is given of which there is no greatest, then on the basis of this second principle of generation a new number is created …(2)

In examining Cantor's discussion of the second principle of generation, one is unable to find a basis for actualizing the infinite (in the illustration, the appearance of the transfinite number, $\omega$, beyond the finite sequence, $v$) other than the mere existence of the *un*actualized, that is, potential infinity. The effect is a tacit assertion that if there is an open infinity, this of itself is the necessary and sufficient condition for its closure upon the transfinite! Does this not amount to *invoking* the actually infinite, defining it into being by fiat?

There can be no denying that to attain the infinite, we must "step out of the series"; there will be discontinuity in the given number field. But discontinuity alone, an absolute discontinuity, signifies nothing more than an incompleteness in understanding. The challenge is to discover the *deeper* continuity operating to engender the infinite. The continuous transformative process by which the infinite is realized must be brought to light.

For the time being, I would like to point out that the problem of generating infinity is not genuinely addressed in conventional mathematical analysis, that is, the standard calculus with its method of limits. But we should be able to see better how the issue is circumvented there, by first considering the relative nature of the infinite.

## 2. THE FIRST PRINCIPLE OF THE RELATIVITY OF THE INFINITE

The relativity of the infinite is implied in Cantor's conception of the transfinite number, for Cantor emphasized time and again that such numbers are not vague but "definite", a term consanguineous with "finite". It is precisely because we ordinarily take the infinite to be absolute that our minds boggle at the notion of a number that is "infinite" yet "definite". The difficulty is resolved as soon as we qualify our use of language. The transfinite number, $\omega$, is infinite *relative* to the lower-order sequence, 1, 2, 3, . . ., $v$, . . . Yet in relation to the sequence of which *it* is a member, $\omega + 1$, $\omega + 2$, . . ., $\omega$ + r, . . ., $\omega$ should be regarded as *finite*. Mathematician Charles Musès makes the same case from a geometric perspective:

> Now it is known that in a given n-dimensional space, the operative presence of an extra or higher dimension that is not suspected, can reflect itself effectively in the mathematics of the lower dimensions as an infinity. Thus a finite unit square (second dimension or $D_2$) may be considered as an infinitely long line.., finite area ($D_2$) reflects as an infinity when only a linear continuum ($D_1$) is considered. Similarly, finite volume ($D_3$) when restricted to only two dimensions may appear as infinite area ($D_2$). Thus, in general, we have the mapping $D_{n+1} \rightleftharpoons \infty\ D_n$, where the left-hand member is some finite portion of (n+1)-dimensional space.

Given the domains of analysis n and n+1, given that n+1 is viewed as an open infinity from within the constraints of n, and given that the transition from n to n+1 closes the infinity, finitizes that which was infinite: How does the classical method of limits operate? What does it accomplish?

The method of limits does *not* provide the transition from n to n+1, does *not* close the open infinity, does *not* finitize that which was infinite. For the method proceeds not from below, but from *above*, as it were. In this regard, Cantor spoke of the relationship between the lower-order series, $a_v$, and the transfinite number, b:

> It is not at all true that the number b is defined as the "limit" of the numbers $a_v$ of a fundamental series ($a_v$). This would be a logical mistake . . . one need not first *obtain* it [i.e., the number b] by a limiting process but on the contrary—through its *possession* one is convinced of the feasibility and evident admissibility of the limiting processes.(2)

So the standard method begins with the infinite already in hand, that is, finitized. Geometrically speaking, we may say that the process commences from the *pre*-actualized, higher-dimensional regime and works backwards from there to the lower domain. By following this procedure, an inter-dimensional analysis is performed which yields a quantitative description of motion or change.

The simplest application is illustrated by the ancient philosophical problem of the dichotomy posed by Zeno. A runner is to traverse an 80 meter race course. Before reaching his objective, he must travel 40 meters. Similarly, to gain 40 meters he must move 20, to advance 20 he must pass through the 10 meter point on the continuum that separates him from his goal, and so on. If this progression is continued, we find the runner unable to take even a single step towards the finish line, much less approach and cross it.

Zeno's paradox of the dichotomy suggests the impossibility of motion. It was founded on the assumption that a line is infinitely divisible, an infinity of points lying between the runner and any other location. However close is point P to P', this mathematical presupposition appears to create the effect of a void. The proximity of P and P' may be arbitrarily increased, but points will not converge.

We recognize the difficulty at once. The Zenoan infinity is a *potential* infinity and we have seen that this potential can never be actualized while continuing to operate numerically within the lower order of finitude. Naturally, in practice, the runner *can* traverse the course, just as we can draw a line on a sheet of paper by extending a point. The standard analysis of this point-to-point motion begins when the motion is a *fait accompli*, that is, it begins from the *higher* order of finitude, namely the line segment that connects the points. However short this finite interval (and proportionately, the time it took to draw it), it will be the completed expression of the total change in position that has taken place. But by the mathematical procedure of *differentiation*, we can obtain the *derivative* of position, that is, the *instantaneous* rate of change in position from which the completed motion derives. Differentiation is achieved by the *method of limits*. For the finite condition, change in position is termed $\Delta Y$ and is taken as a function of time elapsed, $\Delta X$. By taking $\Delta X$ to be arbitrarily small so that it approaches zero (the infinitesimal) as a limit, the instantaneous expression of change in position with respect to time is:

$$dY/dX = \lim_{\Delta x \to 0} \Delta Y/\Delta X$$

Though $\Delta X$ converges upon zero, the *ratio* dY/dX will have a finite value. Therefore, conceptually, the derivative may be regarded as a finite expression of the higher dimension ($D_1$) operating within the lower dimension ($D_0$). It is in this sense that the method of limits "finitizes the infinite", for we have seen that from the strict standpoint of the lower dimension, the higher dimension would appear as an infinity.

Thus we may say that motion transpires in the domain *between* dimensions—between line and point in the case of linear functions—and the method of limits, converging backward from the higher order of finitude, provides determinate access to this domain. The derivative, by revealing the working of the line within the point-position, gives a precise account of movement from point to point. (Of course, higher-order derivatives may be obtained to account rigorously for more complex forms of movement.)

Historically, the standard calculus was highly successful since it was an indispensable tool for the analysis of motion upon which the Newtonian universe was built. The method purportedly deals with the problem of infinity, but, to summarize what I have attempted to indicate in this section, its viability is wholly dependent on the prior, thoroughgoing relativization of the infinite. One does not *obtain* the infinite by the method of limits, to cite Cantor's thinking again. The method can only succeed when one is operating in a domain where the infinite is already at one's disposal, has already been finitized. It is within such a domain that the *first principle* of the relativity of the infinite applies. Here we are dealing with a weakened or *absolutely* relative infinity. The conventional mathematical procedure does not come to grips with the question of *generating* infinities, the problem of building bridges between dimensions. It merely uses the bridges already erected.

## 3. MODERN PHYSICS AND THE FAILURE OF THE FIRST PRINCIPLE OF THE RELATIVITY OF THE INFINITE

Now let us suppose that a bridge has *not* been erected. Suppose that in attempting to describe a dynamic pattern with respect to a given space, n, there is no recourse to a completed, already finitized n+1 regime. Then it would not be possible to show the instantaneous working of said pattern in space n. A finite derivative of the dynamism could not be obtained; the motion with respect to space n would be irremediably divergent, manifesting itself as an infinity. Naturally, determinate analysis would be ruled out. Analytical continuity would yield to fundamental discreteness, since the dynamism would be undifferentiable, indescribable as a continuous function. In such a case, Zeno's kind of paradox, presumably solved once and for all by the method of limits, would find new life, for here infinity would be powerfully revitalized.

This is precisely the dilemma that has arisen in the twentieth century, with science's attempt to penetrate the realms of subatomic process. In the microworld, there is primary indeterminism, inherent discreteness—the problem of infinity returns with a vengeance. Mathematician Musès asserted that the troublesome infinities which turn up in quantum theoretic computations are "the predictable result of omitting a higher dimension," and he cautioned that "'higher dimension' here means not merely a space dimension but a *new* dimension."[(3)] The epistemological significance of Musès' distinction cannot be overestimated. In the language of the present exposition, a mere "space dimension" is one to which a bridge already exists, one which is at the disposal of the analyst, an infinity that has been closed or finitized with respect to the analyst's perceptual/cognitive repertoire. This is what makes determinate analysis possible.

By contrast, the profound epistemological challenge posed by the "dimension" of quantum phenomena derives from its "newness", that is, its *embryonic* status. I propose we regard it as a space that has not been fully incubated, an as-yet unbridged infinity, the bridging of which would entail a transformation of the analytical repertoire. This interpretation does not seem an unreasonable alternative to viewing the microworld as a fully fledged space, which clearly it is not, or merely declaring it a "non-space", which is tantamount to proclaiming its non-interpretability. Moreover, the suggested interpretation is supported in at least a preliminary way by the meta-theory of dimensional generation, independently adumbrated by Musès(4) and, through the medium of intuitive geometry, myself.(5) (The theory is currently under development and will be briefly considered in the concluding section of this paper.)

Nevertheless, most working physicists feel it is unnecessary to interpret the quantum domain in this fashion, indeed, to interpret it at all! If the issue is pressed, they can scarcely deny the extraordinary character of that regime, but the prevailing attitude is that speculation on the nature of quantum reality is best left to the philosophers. Of course, for the program of quantum physics to be carried out, a description of the subject matter is necessary, a formalism must be chosen. The impression is created that the mathematical systems developed to account for quantum processes are pure abstractions devoid of interpretive content. Yet this is contrary to the fact. Far from being interpretively neutral, the algebraic structures of extant quantum field theory rest upon a definite assumptive base and the ground selected is the *classical* one. The underlying presupposition is that quantum space possesses the same basic character as the already finitized infinity, a completed, simple continuousness through which we may follow the standard analytic operating procedure, employ the conventional calculus. Is there not a contradiction here? How can essentially non-classical phenomena be treated in classical terms? By "proper extrapolation to meet the occasion", we are given to believe.

Consider a central feature of the quantum theoretic formalism: analysis by probability. The probabilistic program does not furnish a positive alternative to classical determinism; on the contrary it institutionalizes its failure. The failure of determinism means the failing of simple mathematical continuity, as philosopher Milič Čapek demonstrated in advancing his argument that the constructs of quantum physics raise serious "doubts about spatiotemporal continuity".(6) (Henceforth, when the term "continuity" appears without being qualified, it will be understood to mean *simple* continuity of the analytic kind. It is true that more general forms of continuity exist, as I intimated in speaking of "deeper" continuity earlier in this paper. While sustained attention to this issue would make the scope of the present exposition unmanageable, I will revisit the matter before I am finished.) To Čapek, it was obvious that "the concepts of spatial and temporal continuity are hardly adequate tools for dealing with the micro-physical reality."(6) But it is just this continuity that the probabilistic approach seeks to retain and the price paid is exorbitant. Take, for example, the inability to fix the position of a particle in microspace. Habits of macroworld thinking make it difficult to entertain the explanatory proposition that the microworld particle is just not simply located (that is, it does not occupy a single position in space at a given moment), which would imply that microspace is not simply continuous (non-locality is incompatible with differentiability, a defining property of the simply continuous space). Instead, a *multiplicity* of simply continuous "spaces" is axiomatically invoked to account for the "probable" positions of the particle – "it" is "here" with a certain probability or "there" with another.

Now Charles Musès has cautioned about playing "superficial games with axioms, assuming any self-consistent set that we please". Citing Kurt Gödel's defeat of the formalist's program (spearheaded by David Hilbert) to establish total self-consistency in systems that are devised arbitrarily, Musès says: "We cannot simply invent axiom games at will." It is Musès' conviction that such exercises in what he calls "sterile abstractionism" do not "go unpunished by contradiction".(7) We may ask accordingly, is simple continuity truly preserved by the method of formalistic invocation?

Each subspace of the multi-space expression (the "Hilbert space") is made simply continuous within itself to uphold the mutual exclusiveness of the alternative positions of the observed particle. Such subspaces must be *disjoint* with respect to *each other*, their unity being imposed externally, by fiat, rather than being of an internal, intuitively compelling order. Thus, in the name of maintaining mathematical continuity, a rather extravagantly *dis*continuous state of affairs is actually permitted in the standard formalism for quantum physics, an indefinitely large aggregate of essentially discrete, disunited spaces.

Of course, this is just another way of indicating the impotence of classical analysis in the face of revitalized infinity. The quantum level non-locality in space n may be viewed as infinite motion in n and this, in turn, can be regarded as symptomatic of the higher, n+1 motion upon n for which a finite derivative is unobtainable by our customary procedure. That procedure would set simply continuous space against discrete existence so as to analytically reduce (differentiate) it. But, to reiterate the proposed interpretation of the problem, the requisite order of simple continuity is not at the analyst's immediate disposal, for $D_n$+1 is a "*new* dimension", not a mere space dimension. Confronted with novelty and intent on avoiding it, artificial means may be invented to mask its appearance. Thus, we have the "Hilbert-space" invention, the officially sanctioned stratagem for side-stepping a classically irreducible spatial infinity that implies novel dimensionality.

Physicist Steven Bardwell commented that "the failure to deal with built-in discreteness is the source of the manifold contradictions in quantum field theory".(8) Having examined one aspect of the quantum theoretical contradiction (arising when space would be set against discrete existence, that is, when discreteness would be spatially absorbed by supposing a simply continuous quantum space) we turn to the second. Here focus is on the classically continuous functions of $D_n$. The hope would be to *bypass* the irremediable discreteness indicative of $D_n$+1, but we find to our dismay that we cannot — discrete existence effectively and implacably sets itself against space. Bardwell identifies this as the case "where the relation between the discrete properties that a source introduces into an otherwise continuous field quantity can only be postulated."(8) For instance, when one tries to solve the quantum field equations so as to determine the interaction of an electron with its own electromagnetic field, this "self-energy" value of the electron turns out to be infinite. Physicists generally acknowledge that the appearance of such infinities gives clear evidence of the inadequacy of the standard formalism. Of course, an infinity must be "finitized" if theoretical parameters are to be recoverable by laboratory observations. In the absence of a mathematically natural program for carrying this out, the task is performed by "brute force", by the purely ad hoc calculational procedure known as "renormalization".

Thus, from this second perspective, we again see the primacy of discreteness, of infinity, in the microworld. In this realm, continuity cannot exist indifferently in the face of discrete elements imposed from without, any more than discreteness can be legitimately suppressed by a continuity that is formalistically invoked. Perhaps the problem is more obvious when discreteness is allowed to express itself in the particulate mode, the vantage point from which we witness the destruction of otherwise continuous space by particulate infinity. Nevertheless, from the other perspective, where the attempt is made to absorb discreteness into spatiality, we find discreteness arising from within as the spatial infinity develops.

## 4. THE GENERAL THEORY OF RELATIVITY AND THE PROSPECT OF A UNIFIED CONCEPT OF NATURE

In the twentieth century attempt to account for the physical world in a complete fashion, the quantum physics of the microworld is complemented by a relativistic physics of the macroworld. 0f course, a genuine unification of these approaches has long been sought and still eludes us. Now the problem of infinity that disrupts quantum physical analyses has its counterpart in Einstein's general theory of relativity. It can be shown that the infinities in question in these two cases are identical. I am not merely suggesting that infinities of the same type arise in domains that are remote, domains at opposite extremes on the scale of magnitude. Rather, the point is that the appearance of such infinities signifies a *breakdown* in that scale, revealing the purportedly separate regimes to be one and the same. It follows that if infinity were approached in the proper manner, the unification desired would be achieved. Indeed, the *im*proper strategy of arbitrarily denying resurgent infinity so as to retain analytic continuity is one that *preserves* scale distinctions, thereby preventing anything but a superficial unification of microworld and macroworld.

Philosopher Milič Čapek implied that Einstein's theories of relativity point toward a loss of analytic continuity, this result being manifested most clearly in the general theory. As Čapek would put it, a "dynamization of space" is involved.[(6)] Classical analysis may be described as a procedure for reducing the dynamic to the static. This is the essence of mathematical derivation, as we have seen. Motions, actions and interactions, dynamic patterns that would otherwise express themselves as infinities, uncontrollable discontinuities, can be mathematically derived, expressed in static form, because the space in which they occur is itself simply continuous. Thus, we speak of transformations in space, but never the transformation *of* space, for space is classically pictured as an immutable, inert, three-dimensional container for *lower*-order dynamics.

Yet the transformation *of* space is precisely what the general theory of relativity entails. The theory meets the need to account for non-rectilinear, accelerative motion—identified with gravitational effects—in a relativistic manner. In so doing, Einstein found it necessary to depart from the standard Euclidean formulation and introduce the notion of *spatial curvature*. Gravitation is to be understood as the curvature *of* space, not as the product of forces acting *in* space, where space itself would remain impassive, wholly retaining its simply continuous character. Though the loss of simple continuity is suppressible as long as the curvature remains finite, when the role of curvature is fully played out, the consequence for analytic continuity becomes obvious. Solutions to the field equations for general relativity that predict *infinite* curvature indicate a complete failure of simple continuity.

Now the radical discreteness appearing in general relativity can be said to have a second aspect, for the theory additionally incorporates the idea that gravitational mass is equivalent to inertial mass. Again Čapek's examination provides a helpful beginning. He observed that in pre-Einsteinian physics, inertial mass is interpreted as "the material substance itself . . . [the] core of matter . . . the substantial nucleus of matter", whereas gravitational mass can be viewed as matter's "action on space".(6) Had Einstein completely succeeded in reducing inertia and gravity to simply continuous field expression, they would have been made equivalent in the sense of being simply identical. The distinction between them would have been abolished. But in light of the irrepressible *discreteness* that develops in the general theory, perhaps we can reinterpret the equivalence relation as being more akin to the sort found in the quantum physical context, a Bohrian complementarity in which inertia and gravity are indeed aspects of the same reality but not simply identical.

Have we not already discussed such a dual aspect relation in the context of quantum physical discreteness as such? From one perspective, matter is considered in and of itself, its internal effect on space being disregarded; from the other, matter per se is ignored, and its spatial manifestation takes the foreground (see Fig. 1 below). Then we may associate the infinite gravitational mass of general relativity, that is, the infinite curvature, with the *spatial* infinity of the quantum domain. Here the climactic influence of discreteness is viewed as arising *within* the spatial continuum (Fig. 1a). And the infinite inertial mass of the general theory may be identified with the *particulate* infinity of the microworld, for here the influence of discreteness is seen as imposed upon continuity from without (Fig. 1b). According to the proposed reinterpretation of the principle of equivalence, the spatial and particulate forms of the infinite would be complementary aspects of the same underlying reality.


**Manifestation of matter *within* space** | **space**

**(a)** | **(b)**

**Figure 1. Schema for complementary influence modalities of matter/discreteness upon space/continuity. (a) Spatial/gravitational mode: internal locus of discreteness; (b) Particulate/inertial mode: external locus of discreteness.**

However, the infinities of general relativity deal with large-scale effects, do they not – astrophysical events such as the gravitational collapse of massive stars? Must we not distinguish microworld discontinuity from that which manifests itself at the other end of the scale of magnitude? In fact the contrary is indicated because the appearance of "macroworld" discontinuity signifies a *destruction* of the linearly conceived, Euclid-based scale, as we shall see shortly.

Einstein himself resisted the radical consequences of his theory, for while he was introducing a fundamentally non-classical idea, he did not wish to relinquish the analytic power of the classical approach. So he adopted essentially the same stratagem as his quantum physicist colleagues and paid the same price of self-contradiction. Einstein presupposed that non-Euclidean effects are not found at the micro-level, that is, that spatial curvature, incipient discontinuity, vanishes in the small. Locally, Einsteinian space is homeomorphic to Euclidean space. By assuming Euclideanness-in-the-small, Einstein could write equations using a formalistically elaborated version of the conventional calculus, equations designed to reduce the basically non-classical, non-Euclideanness to classical terms. This amounted to an axiomatic invocation of simple continuity in the face of discontinuity so as to transform away the latter. We may study the attendant self-contradiction from the standpoint of its progressive development.

Under Einstein's classical assumption about the microworld, mass and spatial volume each approach zero as a limit. What does the notion of magnitude consistent with this assumption lead us to expect when we proceed up the scale? Increases in mass should be associated with proportionate increases in volume. Bodies of considerable mass should occupy considerable space and the condition of infinite massiveness would be obtained only upon realizing infinite volume.

The general theory of relativity actually predicts the opposite. When we advance "up" the scale of magnitude, the volume of space does not increase in proportion to mass. This is because mass is associated with the curvature or *de-Euclideanization* of space which, in turn, implies a germinal loss of spatial continuity. We need to understand that mathematical continuity means infinite *spatial density* (which is inversely proportional to the density of matter *in* space): an indenumerable infinity of points lies between any two points (however close) in such a totally "filled in" space. To curve space is to lessen its density. This produces a relativistic effect of scale contraction. The effect is small in the middle ranges of magnitude but becomes quite conspicuous in the presence of great massiveness. Ultimately, at the "scale extreme" of infinite massiveness where the classical requirement is of infinite volume, the volume of space is zero because the density of space is zero. Space has gone singular; a "hole" has appeared. Euclidean space is demolished or, in the language of physicist David Bohm[9], space has become "enfolded".

In view of the fact that the general theory of relativity ends in a subversion of the classical concept of scale, it is obviously inappropriate to characterize gravitational collapse as a macrophysical as opposed to microphysical phenomenon. On the contrary, we must say that gravitational collapse constitutes a *nullification* of this distinction. Only the deeply ingrained habit of Euclidean thinking disposes us to regard the relativistic gravitational effect as "macrophysical". It is so compellingly natural for us to consider extreme massiveness as a large-scale phenomenon that we lose sight of what the radical de-Euclideanization of space actually accomplishes: it returns us to the microworld of quantum physics, the problematic domain allegedly avoided by Einstein!

The crisis besetting Einstein's theory—articulated by physicist Brandon Carter[(10)] for instance, in the doubts he raised that the theory could survive its prediction of gravitational collapse—is that its end result implies a radically non-Euclidean microstructure that rudely contradicts its initial premise of an entirely Euclidean one.

What we are witnessing again is the futility of attempting to contain resurgent infinity. The fate of the analytic scheme dependent upon the impotence of infinity is sealed in the domain where infinity is revitalized. In this connection, researcher Don Reed observed that: "the equations [predicting "black holes", that is, gravitational collapse] are . . . consistent with the conventional . . . (classical) conception of infinity . . . [but that] *Black hole 'infinity' is foreign to any current conceptions of infinity that we are familiar with, involving a higher level of reality 'beyond' space and time*."[(11)]

The crux of Einstein's difficulty, then, was an inability to handle this "black hole 'infinity'", the self-same irreducible infinity currently confronting the quantum physicists. Clearly, as long as efforts to deny the dramatically non-Euclidean character of modern physical phenomena continue, a properly unified conception of nature will be unattainable.

Physicist Arthur Eddington, for one, was quick enough to admit that the curvature cannot really be denied. He identified the microphysical curvature with the "scale uncertainty". His advice: disregard it in favor of the conventional flatspace approach to the quantum regime "so that there is no loss of rigor".[(12)] Eddington's trepidations are as understandable as Einstein's, for once fundamental curvature is accepted, the tool physicists have relied on for centuries—the standard calculus with its method of limits—is no longer of any use. His message then is this: it is better to ignore a micro-level non-Euclideanness that would require developing an entirely new approach, and to proceed as before, though certain "adjustments" do have to be made now, to keep the formalism viable. Yet the fact is inescapable that the necessity for such "adjustments" is tantamount to methodological failure. And it is the choice to continue making them that keeps us from the unity so urgently required.

## 5. TOWARD A SECOND PRINCIPLE OF THE RELATIVITY OF THE INFINITE

In sum, the crisis in modern theoretical physics is a crisis concerning infinity. The infinities of the classical era are a weak, relative, "garden variety", for these, in fact, have already been actualized or finitized with respect to our analytic, perceptual/cognitive repertoire. But they are giving way to a tough new strain, one creating problems and paradoxes for us that appear insurmountable. We may resist, stubbornly cling to the established order of the finite. We may refuse to compromise. But in that case, infinity will not compromise; it will persistently assume an absolute, unmanageable form. Then is there any alternative but to accept the infinite?

Now the quantum-level, "black hole" infinity is certainly not relative in the first sense of the total relativity of the infinite, but if it were simply absolute, accepting it would mean complete capitulation. There would be no choice but to resign ourselves to the end of the scientific enterprise. Ultimately, we would be obliged to surrender our analytic capacities once and for all, having lost our hold on space, time, dimension.

On the other hand, what if the "black hole microspace" could be viewed as a *new* dimension instead of merely a non-dimension? I mentioned this possibility earlier. An *embryonic dimension* would be one that the analyst does not yet have at his or her disposal—an infinity not yet finitized with regard to the analyst's repertoire. An altogether different attitude of acceptance would be called for here. The infinite could be accepted in an *active* manner, as a challenge to be met, for an embryo is a promise of renewal.

Supporting the notion of nascent dimensionality is the idea of *dimensional generation*. This is not an analytic theory, but a meta-theory that we could characterize as "transanalytic". Analytic theories share the assumption epitomized in the Euclidean philosophy of Immanuel Kant: space—and time, a concept admittedly neglected in the present exposition—are a priori forms of awareness constituting a fixed framework that contains and constrains all contingencies and dynamisms. From the classical standpoint, the dynamic must yield to the static. In contrast, the idea of dimensional generation makes dynamic process basic by infusing space and time themselves with historical attributes. A perspective on spatiotemporal dimensionality is suggested from which the classical, analytic view can be seen as but a phase in a developmental context possessing a *pre*-classical origin and a *post*-classical terminus. Beyond even that, the meta-theory provides what we expect it must, as a concept making on-going process fundamental: there is a regeneration of analytic capacity at a more comprehensive level. Thus, the meta-theory is *trans*analytic.

As has been noted earlier, Musès' introduction of dimensional generation and my own, speak to the issue. Also, the work of mathematician Benoit Mandelbrot(13) on "fractals" (related to fractional dimension) furnishes some important insights, as does the more recent attempt of Frescura and Hiley(14) to mathematize Bohm's "implicate order".

I believe that this kind of effort not only lends credence to the proposed interpretation of the quantum dimension as embryonic but may constitute the first step toward bringing the "embryo" to term. Actively accepting the "built-in discreteness", the "irremediably" infinite, means modelling it intrinsically, embodying it without attempting to reduce it, as analytic modelling would. In the *trans*analytic approach, discontinuity is accepted by a natural generalization of algebraic and geometric structure that affords a deeper continuity, an impartible, therefore unanalyzable, unity. Then, when the meta-theory achieves its fullest expression, we see the intimate interplay of deeper continuity with continuity of the simple, differentiative sort, a dialectic through which particular analytic regimes are destroyed and new ones unfold. This transformative scheme is the one foreshadowed in my previous presentations of the "Moebius-Klein" meta-geometry(15-18) and, if I am not misinterpreting him, it is inherent in Charles Musès' algebraic concept of *hypernumber*,(7) the hypernumber *w* playing an especially significant role.

Such radically non-linear meta-modeling of dimensional embryogenesis will be no mere exercise in armchair abstractionism—not if it is inherently valid, not if its profound methodological implications are pursued to their conclusion with intuitive honesty. An ad hoc description, one that is arbitrary and self-serving, will bear little fruit. But a description that faithfully adheres to the "underlying mathematical reality", as Musès (citing Godfrey Hardy) termed it,[7] will have concrete creative consequences. In other words, to describe change in an intuitively coherent manner is to constructively *foster* change.

Here is our intimation of a second, more general principle of the relativity of the infinite. For, while the first principle of the *completely* relative infinity has presently failed us, I am proposing that we can deal with the infinite in its stronger manifestation, "finitize" it, so to speak, divest it of its absolute character. But not by beating it into submission, not by establishing supremacy over it from an entrenched position in our home territory. We must meet it halfway, that is, be willing to relinquish the *established* order of finitude, admit elements of potentiality that will transmute and expand what was already actualized. The essential idea is enunciated by Francisco J. Varela in his discussion of paradoxes. "Instead of finding ad hoc means of *avoiding* their appearance (as in Russell's theory of types) we let them appear freely . . ."[19] To illustrate, Varela draws from the history of arithmetic. Mathematicians were able to extend the real number domain to that of the complex numbers by accepting the idea that a number (namely, the "imaginary" number i) may be both positive and negative. "Again, rather than avoid the antinomy, by confronting it, a new domain emerges."[19]

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